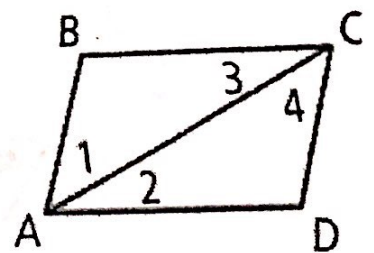
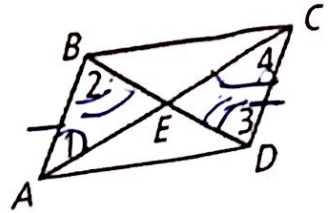


| | | |
|-----------|---|--|
| Sides | A parallelogram is a quadrilateral with both pairs of opposite sides parallel. | |
| | If a quadrilateral is a parallelogram, the 2 pairs of opposite sides are congruent. | |
| Angles | If a quadrilateral is a parallelogram, the 2 pairs of opposite angles are congruent. | |
| | If a quadrilateral is a parallelogram, the consecutive angles are supplementary. | |
| | If a quadrilateral is a parallelogram and one angle is a right angle, then all angles are right angles. | |
| Diagonals | If a quadrilateral is a parallelogram, the diagonals bisect each other. | |
| | If a quadrilateral is a parallelogram, the diagonals form two congruent triangles. | |

Example 1: Given: $\square ABCD$ is a parallelogram.
Prove: $AB = CD$ and $BC = DA$.

| Statement | Reason |
|--|---|
| 1. $ABCD$ is a parallelogram | 1. GIVEN |
| 2. $AB \parallel BC$ | 2. Definition of a parallelogram |
| 3. $\angle 1 = \angle 4, \angle 3 = \angle 2$ $AC = AC$ | 3. Alt. int. \angle's Thm |
| 5. $\triangle ABC \cong \triangle CDA$ | 4. Reflexive Prop |
| 6. $AB = CD$ and $BC = DA$ | 5. Def. Parallelogram |
| | 6. CPCTC |

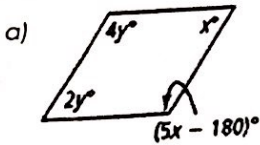




Example 2: Given: $\square ABCD$ is a parallelogram.
Prove: AC and BD bisect each other at E.

| Statement | Reason |
|---|-----------------------------|
| 1. ABCD is a parallelogram | 1. Given |
| 2. $AB \parallel DC$ | 2. Def. parallelogram |
| 3. $\angle 1 = \angle 4, \angle 2 = \angle 3$ | 3. Alt. int \angle 's thm |
| 4. $AB = DC$ | 4. Prop. of parallelogram |
| 5. $\triangle ABE \cong \triangle CDE$ | 5. ASA |
| 6. $AE = CE, BE = DE$ | 6. CPCTC |
| 7. AC and BD bisect each other at E. | 7. Definition of bisector |

Example 3: For what values of x and y must each figure be a parallelogram?



$$x + 5x - 180 = 180$$

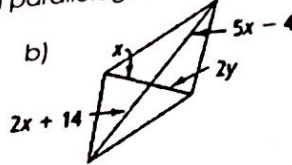
$$6x - 180 = 180$$

$$6x = 360$$

$$x = 60$$

$$6y = 180$$

$$y = 30$$



$$2x + 14 = 5x - 4$$

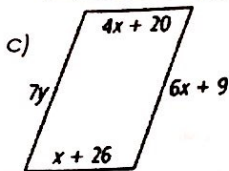
$$14 = 3x - 4$$

$$18 = 3x$$

$$6 = x$$

$$6 = 2y$$

$$3 = y$$



$$4x + 20 = x + 26$$

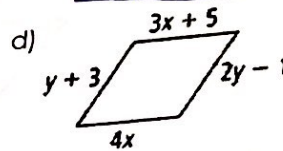
$$3x = 6$$

$$x = 2$$

$$6(2) + 9 = 7y$$

$$21 = 7y$$

$$3 = y$$



$$4x = 3x + 5$$

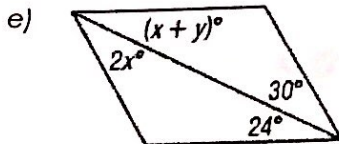
$$x = 5$$

$$y + 3 = 2y - 1$$

$$3 = 2y - 1$$

$$4 = 2y$$

$$2 = y$$

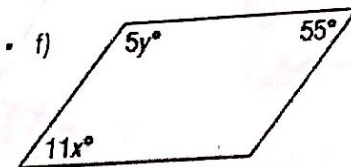


$$2x = 30$$

$$x = 15$$

$$15 + y = 24$$

$$y = 9$$



$$5y + 55 = 180$$

$$5y = 125$$


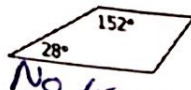
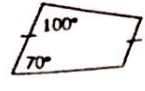
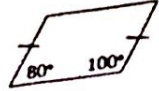
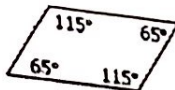
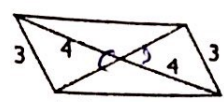
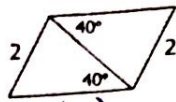
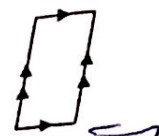
$$y = 25$$

$$11x = 55$$

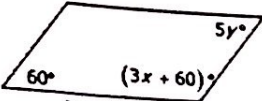
$$x = 5$$

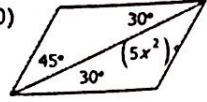
5.3 Proving Parallelograms Practice

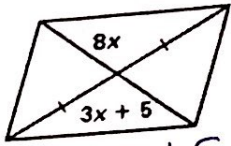
Determine if each quadrilateral is a parallelogram. Explain why or why it does not work.

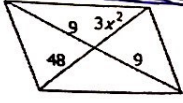
1)  **Yes** ~~Diagonals bisect each other~~
 2)  **No trap**
 3)  **NO**
 4)  **Yes**
 5)  **yes**
 6)  **NO**
 7)  **NO**
 8)  **Yes**

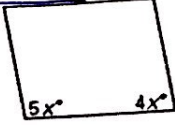
Find the value of x and y that ensure each quadrilateral is a parallelogram.

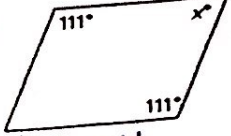
9)  $5y = 60$
 $y = 12$
 $120 = 3x + 60$
 $3x = 60$
 $x = 20$

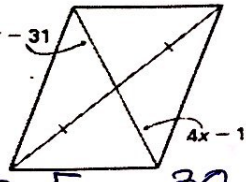
10)  $5x^2 = 45$
 $x = 9$
 $x = 3$

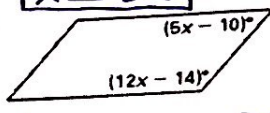
11)  $8x = 3x + 5$
 $5x = 5$
 $x = 1$

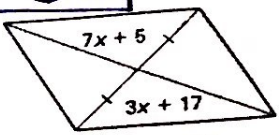
12)  $48 = 3x^2$
 $16 = x^2$
 $4 = x$

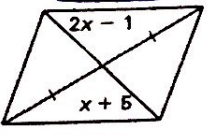
13)  $9x = 80$
 $x = 20$

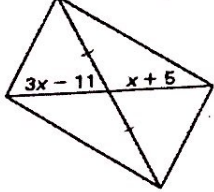
14)  $180 = 111$
 69

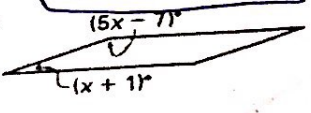
15)  $9x - 31 = 4x - 1$
 $5x = 30$
 $x = 6$

16)  $17x - 24 = 180$
 $17x = 204$
 $x = 12$

17)  $7x + 5 = 3x + 17$
 $4x = 12$
 $x = 3$

18)  $2x - 1 = x + 5$
 $x = 6$

19)  $3x - 11 = x + 5$
 $2x = 16$
 $x = 8$

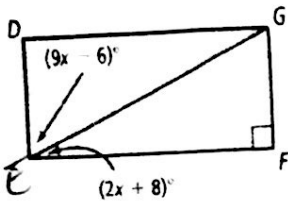
20)  $6x - 6 = 180$
 $6x = 186$
 $x = 31$

5.4 Quadrilaterals

SWBAT use the properties of quadrilaterals to solve for unknowns.

| Rectangle | Rhombus | Square |
|---|---|---|
| A rectangle is a parallelogram with four right angles. | A rhombus is a parallelogram with four congruent sides. | A square is a parallelogram with four congruent sides and four right angles. |
| A rectangle has all the properties of a parallelogram PLUS: 4 right angles Diagonals are congruent | A rhombus has all the properties of a parallelogram PLUS: • 4 congruent sides • Diagonals bisect angles • Diagonals are perpendicular | A square has all the properties of a parallelogram PLUS: • All the properties of a rectangle • All the properties of a rhombus |
| | | |

Example 1: Solve for x and the measure of each angle if $\square DGFE$ is a rectangle.



$$9x - 6 + 2x + 8 = 90$$

$$11x + 2 = 90$$

$$11x = 88$$

$$x = 8$$

$$\angle DEG = 60$$

$$\angle GEF = 24$$

Example 2: $\square ABCD$ is a rectangle whose diagonals intersect at point E .

a) If $AE = 36$ and $CE = 2x - 4$, find x .

$$36 = 2x - 4$$

$$40 = 2x$$

$$20 = x$$

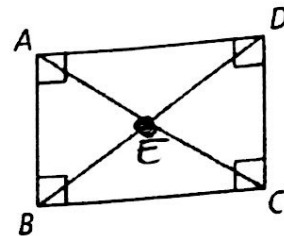
b) If $BE = 6y + 2$ and $CE = 4y + 6$, find y .

$$6y + 2 = 4y + 6$$

$$2y + 2 = 6$$

$$2y = 4$$

$$y = 2$$



Example 3: Using the diagram to the right to answer the following if $\square ABCD$ is a rhombus.

a) Find the $m\angle 1$.

$$90^\circ$$

b) Find the $m\angle 2$.

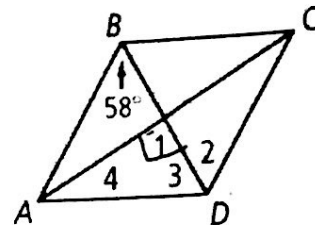
$$58^\circ$$

c) Find the $m\angle 3$.

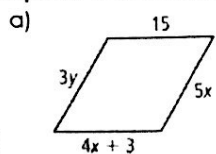
$$58^\circ$$

d) Find the $m\angle 4$.

$$32^\circ$$



Example 4: Solve for each variable if the following are rhombi.

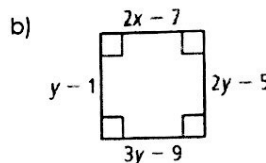


$$15 = 4x + 3$$

$$12 = 4x$$

$$3 = x$$

$$y = 5$$



$$2y - 5 = y - 1$$

$$y = 4$$

$$3(4) - 9 = 2x - 7$$

$$3 = 2x - 7$$

$$10 = 2x$$

$$x = 5$$

Trapezoid

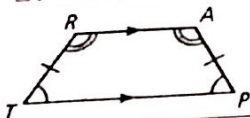
A trapezoid is a quadrilateral with exactly one pair of parallel sides, called bases, and two non-parallel sides, called legs.

Isosceles Trapezoids

An **isosceles trapezoid** is a trapezoid with congruent legs.

- A trapezoid is isosceles if there is only:
- One set of parallel sides
 - Base angles are congruent
 - Legs are congruent
 - Diagonals are congruent
 - Opposite angles are supplementary

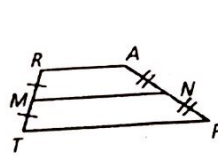
$$\angle T \cong \angle P, \angle R \cong \angle A$$



Trapezoid Midsegment

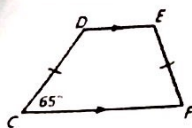
The **median** (also called the midsegment) of a trapezoid is a segment that connects the midpoint of one leg to the midpoint of the other leg.

Theorem: If a quadrilateral is a trapezoid, then a) the midsegment is parallel to the bases and b) the length of the midsegment is half the sum of the lengths of the bases



- (1) $\overline{MN} \parallel \overline{TP}$, $\overline{MN} \parallel \overline{RA}$, and
- (2) $MN = \frac{1}{2}(TP + RA)$

Example 5: CDEP is an isosceles trapezoid and $m\angle C = 65^\circ$. What are $m\angle D$, $m\angle E$, and $m\angle F$?

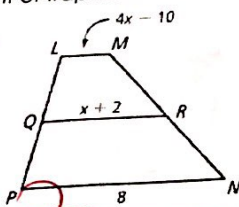


$$\begin{aligned} \angle D &= 115^\circ \\ \angle E &= 115^\circ \\ \angle F &= 65^\circ \end{aligned}$$

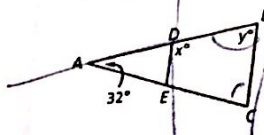
Example 7: QR is the midsegment of trapezoid LMNP. What is x and the length of LM?

$$\begin{aligned} x+2 &= \frac{1}{2}(4x-2) \\ x+2 &= 2x-1 \end{aligned}$$

$$3 = x$$



Example 6: What are the values of x and y in the isosceles triangle below if $DE \parallel DC$?

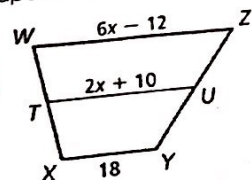


$$\begin{aligned} y &= 74^\circ \\ x &= 104 \end{aligned}$$

You Try! TU is the midsegment of trapezoid WXYZ. What is x and the length of TU?

$$\begin{aligned} 2x+10 &= \frac{1}{2}(6x-12+18) \\ 2x+10 &= \frac{1}{2}(6x+6) \\ 2x+10 &= 3x+3 \end{aligned}$$

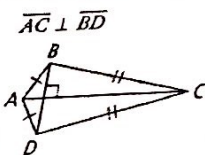
$$7 = x$$



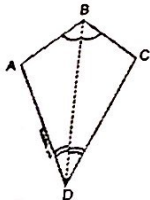
Kite

A kite is a quadrilateral with two pairs of adjacent congruent sides.

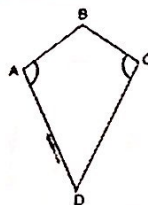
Its diagonals are perpendicular.



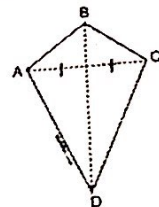
Its diagonals bisect the opposite angles.



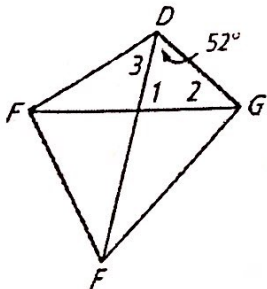
One pair of opposite angles are congruent.



One diagonal bisects the other.

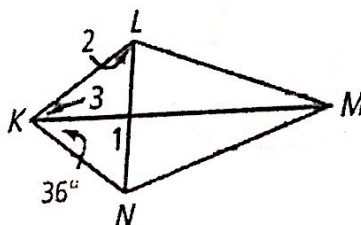


Example 4: Quadrilateral DEFG is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?



$$\begin{aligned} \angle 1 &= 90^\circ \\ \angle 2 &= 38^\circ \\ \angle 3 &= 52^\circ \end{aligned}$$

You Try! Quadrilateral KLMN is a kite. What are $m\angle 1$, $m\angle 2$, and $m\angle 3$?



$$\begin{aligned} \angle 1 &= 90^\circ \\ \angle 2 &= 54^\circ \\ \angle 3 &= 36^\circ \end{aligned}$$