Solving Exponential Equations with the Same Base

Property of Equality for Exponential Equations

If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Ex. 1 $9^{2x} = 27^{x-1}$	**Ex. 2** $100^{7x} = 1000^{3x-2}$
Ex. 3 $4^x = (\frac{1}{2})^{x-3}$	**Ex. 4** $81^{3-x} = (\frac{1}{3})^{5x-6}$
Ex. 4 $\sqrt{5} = 25^{x-1}$	**Ex. 5** $8^{x-1} = 32^{3x-2}$
Ex. 5 $3^x = \frac{1}{81}$	**Ex. 6** $3^x = 9\sqrt{3}$
Ex. 7 $5^x = 5\sqrt{5}$	**Ex. 7** $4^{2x} = 16\sqrt[3]{4}$

Day 2: Exponentials

Exploring Exponential Models

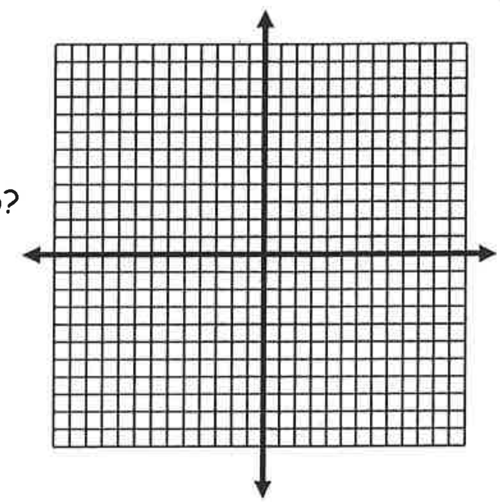
Standard Form of an Exponential Function

- The standard form of an exponential function is ______________________________
- This can also be written as ____
- a and b are constants. "a" is the _____________________ of the function or the ____________________
- "b" is the _____________ or _____________ factor

For a > 0

Example: $y = 2(3)^x$

- If ____________, the function models ___________
- "b" is the growth factor.



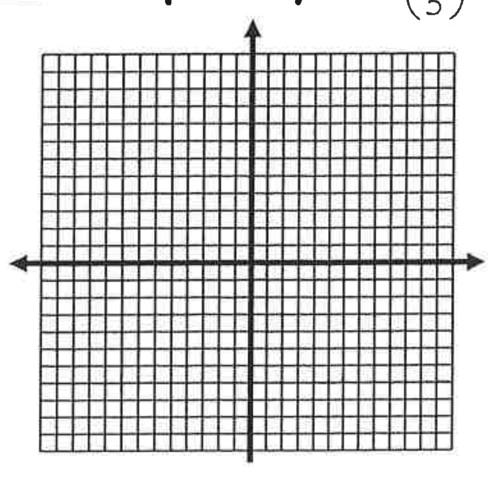
- Will the value of y ever equal zero?
- What is the domain? Range?

Domain:

Range:

- If _______________, the function models __________
- "b" is the decay factor

Example: $y = 1\left(\frac{1}{3}\right)^x$



What is the domain? Range?

Domain:

Range:

Example 0.5 Name the transformations of $y = 2^x$. Then, give the domain and range.

A. $y = 2^x + 7$ B. $y = 2^{x-3}$ C. $y = 2^{x+4} - 9$

Example 1: The population of the U.S. in 1994 was about 260 million, with an average annual rate of increase of about 0.7%

- What is the growth factor for the U.S. population?
- Suppose this rate of growth continues. Write an equation that models the future growth of the U.S. population.
- Predict the population in the U.S. in the year 2001.

Example 2: Suppose the population of a certain endangered species decreases at a rate of 3.5% per year. You have counted 80 of these animals in the habitat you are studying.

- Write the equation for the population each year.
- Predict the number of animals that will remain after 10 years.
- At this rate, after how many years will the population first drop below 15 animals?

Example 3: A car that 5 years ago cost $20,000, is now worth only $13,500. What is the average annual rate of depreciation?

Example 4: Given the function $y = 5(1.61)^x$, state whether the function represents growth or decay and state the percentage rate of increase or decrease.

Example 5: Given the function $y = 3(.56)^X$, state whether the function represents growth or decay and state the percentage rate of increase or decrease.

I. Half Life

- The half life of a substance is the time it takes for half of the material to ____________
- A 3000-mg sample of a certain radioactive element has a half life of 3 seconds. How much of the sample remains after 1 minute?

- Arsenic-74 is used to locate brain tumors. It has a half life of 17.5 days. Write the exponential decay function of a 90-mg sample. Use the function to find the amount remaining after 6 days.

II. Compound Interest

- The compound interest formula for the amount A in an account is

____

P = ______________ r = ______________

n = ______________ t = ______________

- Jodie's parents started a savings account for her when she was born. They invested $500 in an account that pays 6% interest compounded annually. Find the balance of the account after each of the first three years.

- Graham's grandparents started a savings account for him when he was born. They invested $100 in an account with 8% annual interest compounded quarterly. How much is in his account on his 16^{th} birthday?

Logarithmic Functions Notes

Logarithm: In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, y is called the ***logarithm*** of x. It is usually written as $y = log_b\, x$ and is read *"y equals log base b of x."*

**The inverse function of the exponential functions with base b, is called the logarithmic function with base b. For $x > 0, b > 0, b \neq 0$,

$$b^x = y \Longrightarrow x = \log_b y$$

EXPONENTIAL FORM — LOGARITHM FORM

I. Rewriting in both forms.

Example 1) Rewrite logarithmic each equation in its equivalent exponential form.

a. $\log_5 x = 2$

b. $\log_3 7 = y$

c. $2 = \log_b 25$

d. $3 = \log_b 64$

e. $3 = \log_7 x$

f. $\log_4 26 = y$

Example 2) Rewrite each exponential equation in its equivalent logarithmic form.

a. $12^2 = x$

b. $2^5 = x$

c. $8^3 = c$

d. $b^3 = 8$

e. $b^3 = 27$

f. $4^y = 9$

II. Basic and Inverse Log Properties- Because logs are exponents, they have properties that can be verified using the properties of exponents.

Basic Properties:

1. $\log_b b = 1$ because $b^1 = b$
2. $\log_b 1 = 0$ because $b^0 = 1$

Inverse Properties: (Cancel with the same base!)

1. $\log_b b^x = x$
2. $b^{\log_b x} = x$

Example 3) Evaluate using the log properties.

a. $\log_7 7$

b. $\log_5 1$

c. $\log_4 4^5$

d. $\log_7 7^8$

e. $\log_9 9$

f. $\log_8 1$

g. $6^{\log_6 9}$

h. $3^{\log_3 17}$

II. **Common Logarithm**: Base 10 Logarithm, usually written **without the subscript 10.**
$\log_{10} x = \log x, x > 0$. Most calculators have a [LOG] key for evaluating common logarithms.
The calculator is programmed in base 10.

Example 4) Find the value of each log. Round to the nearest ten-thousandths.

a. $\log 81{,}000$

b. $\log 6$

c. $\log 0.35$

d. $\log 0.0027$

V. **Evaluating Logs using the Change of Base Formula**

For all positive numbers, a, b, and n, where $a \neq 1$ *and* $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Example: $\log_5 12 = \frac{\log_{10} 12}{\log_{10} 5}$

This formula allows us to evaluate a logarithmic expression of any base by translating the expression into one that involves common logarithms.

Example 5) Evaluate each logarithm

a. $\log_3 18$

b. $\log_4 25$

c. $\log_2 16$

d. $\log_{25} 5$

e. $\log_2 1024$

f. $\log_5 125$

V. **Solving for variables with exponentials and logs.**

****MAY HAVE TO REWRITE AND APPLY PROPERTIES OR CHANGE OF BASE FORMULA!!!

Example 6) Solve for the variable:

a. $\log_3 243 = y$

b. $\log_9 x = -3$

c. $\log_8 n = \frac{4}{3}$

Example 7) Evaluate:

a. $\log_8 8^4 = x$

b. $\log_9 9^2 = y$

Example 8) Solve each log equation. Be sure to check your answers!

a. $\log_3(3x - 6) = \log_3(2x + 1)$

b. $\log_6(3x - 1) = \log_6(2x + 4)$

Solving using Simple Logarithms

SWOOSH Method	Change of Base	Log = Log
$\text{Log}_{\#}(x) = \#$	$\text{Log}_{\#}(\#) = x$	Log(x) = Log(x)
Use when a variable is attached to the logarithm.	Use when a constant is attached to the logarithm.	Use when one log is = to one other log. Logs must have the same base in order to cancel.

Example 1: Solving using the SWOOSH Method

a) $\text{Log}_2(2x + 1) = 4$

b) $\text{Log}_4(17x - 4) = 3$

c) $\text{Log}(2x - 5) = 2$

Example 2: Solving using Change of Base

a) $\text{Log}_2 8 = 3x + 3$

b) $\text{Log}_5 125 = x^2 - 2x$

c) $\text{Log}_2 16 = x^2$

Example 3: Solve by canceling the logs!

a) $\log_4(3x-1) = \log_4(2x+3)$

b) $\log_2(x-6) = \log_2(2x+2)$

Properties of Logarithms **Name**______________________________

Objective: To apply the properties of logs to condense and expand logarithms and to use these properties to solve logarithmic equations.

Properties of Logarithms

1. Product Property: $log_b mn =$
2. Quotient Property: $log_b \frac{m}{n} =$
3. Product Property: $log_b m^x =$

Example 1: Condense the following logarithmic expressions using the properties of logs

A. $\log x + \log 2 + \log y$

B. $log_2 x - log_2 7$

C. $2log\, x + log\, y$

D. $\frac{1}{2} log_3 x + 2log_3 y$

E. $\frac{1}{4}\log x - (2\log y - 5logz)$

Example 2 Expand the following the logarithmic expressions using the properties of logs

A. $\log(xyz)$

B. $log_{2\frac{4x}{y}}$

Example 3 Solve the following logarithmic equations.

A. $log_3 5 + log_3 x = log_3 10$

B. $log_2 x + log_2(x+2) = 3$

C. $\log 16 - \log 2x = \log 2$

D. $4log_2 x + log_2 5 = log_2 405$

E. $\log(x+21) + log\ x = 2$

F. $2log_2(x+2) = 10$

Math 3

2.5 Equations without Logs

Unit 2

SWBAT solve equations initially without logarithms by using either similar bases or the properties of logs.

Solving equations with NO logs!

Method 1: Similar Bases
(Note: Does not work for every problem)

Step 1: Isolate the Base

Step 2: Write both sides of the equation as an exponential with like bases.

Step 3: Set exponents equal to each other.

Step 4: Solve for the unknown.

Example 1: $2^{2x+1} = 32^x$

Example 2: $-5 + 5^{3x-9} = 120$

Example3: Solve for x: $3^{2x} = 27$

You Try! Solve for x: $2^x = 8$

Why would you need to use a log? Because the variable is in the ______________ and logs bring them down!!

Method 2: Properties of Logs

Step 1: Make sure the piece with the unknown exponent is ______________ on one side.

Step 2: ______________ the logarithm to each side.

Step 3: Use the __________________ to bring down the exponent and solve!

Example 1: Solve for x: $5^{3x} = \frac{1}{125}$

You Try! Solve for x: $2^{5x+1} = 32$

Example 2: Solve for x: $3^x + 5 = 40$

You Try! Solve for x: $2(6^{2x}) = 20$

The Many Ways to Solve a Logarithmic Equation

One Log	**SWOOSH!** Use when a variable is attached to the logarithm.	Solve for x: $\log_4(4x - 2) = 3$
	Change of Base Use when the variable is <u>not</u> attached to the logarithm.	Solve for x: $\log_2 45 = x$
Two Logs	**Cancel the logs!** Do this if and only if there is <u>one</u> log per side.	Solve for x: $\log_6 x = \log_6 2x - 2$
	Condense the logs So that only one log appears per side. Then, decide whether to cancel, swoosh, or use change of base.	Solve for x: $3\log_2 x + \log_2 5 = 7$
No Logs	**Add a Log!** Use this if you cannot get similar bases.	Solve for x: $7^{x-3} + 5 = 30$
	Similar Bases! Break each base down so that they are the same, cancel the bases, and work only with the exponents!	Solve for x: $25^{2x} = 125$

Practice: Complete the following problems for extra practice using the above rules for solving logarithms.

1. $2\log_4 x = 12$
2. $\text{Log } 5x - \log 7 = 2$
3. $\log_5 15 = 3x$
4. $4^{3x} \cdot 4^{2x} = 1048576$

Natural Logarithms and Base e

The Natural Base ______ is an irrational number and is approximately ___________. It is often called ___________ number.

Ex. 1 Simplify natural base expressions

A. $e^2 \cdot e^5$	B. $\frac{12e^4}{3e^2}$	C. $(5e^{-3})$^2	D. $5e^{-3} \cdot 2e^2$

Ex. 2 Use your calculator to evaluate Natural Base Expressions

A. $e^{0.5}$	B. e^{-8}	C. e^2

Natural Logarithm is a logarithm with base _________.

The Natural logarithm function is ____________________.

Example 3 Evaluate Natural Base Expressions

A. $\ln 3$	B. $\ln \frac{1}{4}$	C. $\ln 0.05$	D. Ln e

Example 4 Simplify the expression

A. $\frac{\ln e^4}{8}$	B. $\ln e^{8.3}$	C. $10 \ln e$	D. $\ln 1$

Example 5 Write each as a single logarithm. Use the properties to condense.

A. $3ln\,5$

B. $ln\,24 - ln6$

C. $\frac{1}{3}(lnx + lny) - 4lnz$

D. 2 ln 8 -3 ln 4

Example 6 Solve Base e Equations   Remember….Isolate the e. Then take the ln of each side.

A. $e^{\frac{x}{4}} + 3 = 9$

B. $5e^{-x} - 7 = 2$

C. $e^{3x+1} = e^{13}$

Example 7 Solve Natural Log Equations

A. $ln5 - lnx = 4$

B. $ln(2m + 3) = 8$

C. $ln\frac{x-3}{4} = 8$

Applications of Natural Logs and Base e

***To calculate continuously compounded interest, we use the formula:

y = 　　　　　　r =

P = 　　　　　　t =

Example 6: How much money will be in a bank account after 1.5 years if you invested $400 at 7.6% compounded continuously?

Practice: Complete the following problems for class work. Show all work.

1. Solve $\ln(14x - 3) = \ln(7x + 11)$
2. Solve $2e^x - 5 = 1$
3. $\ln(x - 1) = -2$
4. $\ln(2x - 3) = 2.5$
5. $\ln 48 - \ln x = \ln 4$
6. $e^{3x} \cdot e^x = 15$

Mixed Review: Remember, *all* logarithms share the same rules. Always condense first before solving!

7. $4^{3x} = 12$
8. $\log_6 x + \log_6 9 = \log_6 54$
9. $\log_2 x = -3$
10. $\log_2 64 = x$
11. $\log_2 x - \log_2 5 = 3$
12. $\ln 4x + \ln 5 = \ln 20$
13. Mazie invested $4500 in an account earning 4.3% interest compounded continuously. After how many years will she have $7400 in her account?

Math 3

2.7 Graphing Exponentials and Logs

Unit 2

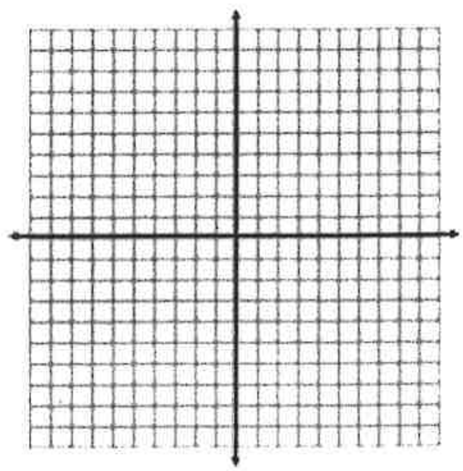
SWBAT graph exponential and logarithmic functions on the coordinate plane.

Exponential Function		Logarithmic Function	
A function whose unknown (x) is located in the exponent		The inverse function of an exponential function.	
Transformations: $y = a(b)^{(x-h)} + k$		**Transformations:** $y = a \cdot \log_b(x-h)+k$	
Domain:		**Domain:**	
Range:		**Range:**	
Asymptote:		**Asymptote:**	

Example 2: Graphing Exponential Functions

a) Graph $y = 2^{x+3} - 5$

x	y



Transformations:

Asymptote:

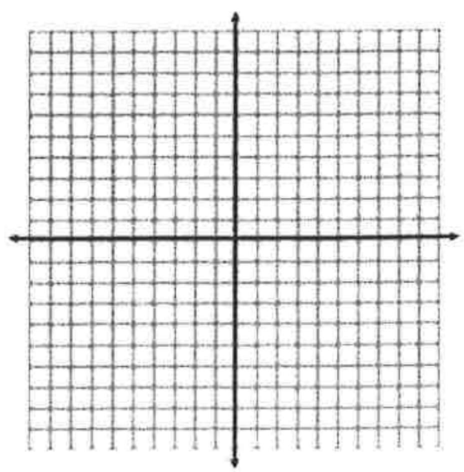
Domain:

Range:

End Behavior:

b) Graph $y = -3^{x-1} + 6$

x	y



Transformations:

Asymptote:

Domain:

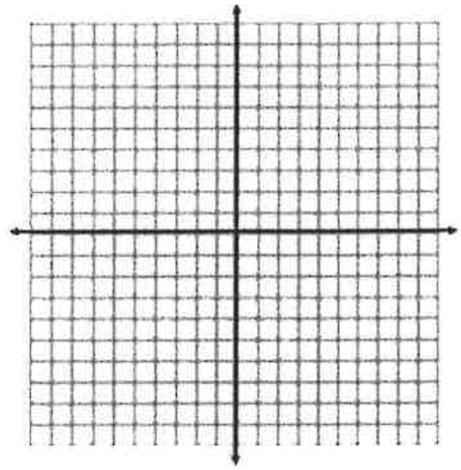
Range:

End Behavior:

Example 3: Graphing Logarithmic Functions

a) Graph $y = \log_2 x - 3$

x	y



Transformations:

Asymptote:

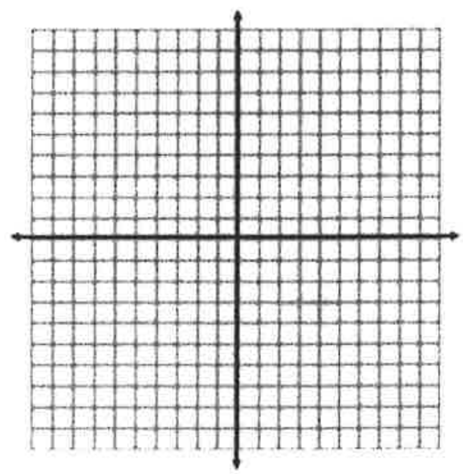
Domain:

Range:

End Behavior:

b) Graph $y = -\log_4(x + 4) + 2$

x	y



Transformations:

Asymptote:

Domain:

Range:

End Behavior: