# CONCEPT #1: Direct & Inverse Variation

1. When pairs of numbers $(x, y)$ have a ***ratio*** that is constant, we say that there is _____ variation.

2. When pairs of numbers $(x, y)$ have a ***product*** that is constant, we say that there is _____ variation.

3. Create a table of values where the constant of variation $k = 2.4$ and the pairs of numbers $(x, y)$ vary directly.

4. Create a table of values where the constant of variation $k = 25$ and the numbers $(x, y)$ vary inversely.

5. Write a function that models the relationship. Z varies jointly with $x$ and $y$ and inversely with $w$. When $x = 5$, $y = 4$, $w = 26$ and $z = 10$.

$$Z = \frac{13xy}{w}$$

6. Use the equation from question 5 to find $w$ if [redacted] $z = 52$, $x = 4$, $y = 19$.

$w = 19$



*Jean Adams*

# Concept 1

(5) $Z = \frac{kxy}{w}$

$10 = \frac{k(5)(4)}{26}$

$10 = \frac{20k}{26}$

$\frac{260}{20} = \frac{20k}{20}$

$13 = k$

$Z = \frac{13xy}{w}$

(6) $Z = \frac{13xy}{w}$

$52 = \frac{13(4)(19)}{w}$

$52w = 988$

$\frac{52w}{52} = \frac{988}{52}$

$w = 19$

# CONCEPT #2:
# Discontinuities, Holes, & Asymptotes

1. Write an equation for a rational function that has a hole at $x = 2$ and a vertical asymptote at $x = -4$.

$y = \frac{(x-2)}{(x-2)(x+4)}$

2. Write an equation for a rational function that has a horizontal asymptote at $y = 3$.

$y = \frac{3x^2}{x^2+1}$

3. Given the graph of $y = \frac{x^2 + 2x - 35}{x^2 + x - 30}$ find the following:

A. Horizontal Asymptote $y = 1$

B. Vertical Asymptote $x = -6$

C. Hole coordinates $(x, y)$ when $x$ is 5

D. $x$-intercept $(x, 0)$ Root $(-7, 0)$

E. $y$-intercept $(0, y)$ $(0, 7/6)$

F. Domain Restrictions $(-\infty, -6) \cup (-6, 5) \cup (5, \infty)$



Jean Adams

# Concept #2

1. $y = \frac{(x-2)}{(x-2)(x+4)}$

2. $y = \frac{3x^2}{x^2+1}$

3. $y = \frac{x^2+12x-35}{x^2+x-30}$ $\quad y = \frac{(x+7)(x-5)}{(x+6)(x-5)}$

A: Horiz. Asymp: [redacted] $y = 1$

B: Vert. Asymp: $x = -6$

C: Hole: When x is 5

D: $(-7, 0)$

E. $(0, 7/6)$

F: $(-\infty, -6) \cup (-6, 5) \cup (5, \infty)$

# CONCEPT #3:
# Operations with Rational Expressions

1. **Multiply, state restrictions on the variables.**

$$\frac{5x-20}{3x+15} \cdot \frac{7x+35}{5x^2-80}$$

$\frac{7}{3(x+4)}$ $x \neq -5, \pm 4$

2. **Divide, state restrictions on the variables.**

$$\frac{y^2-5y+4}{1-y^2} \div \frac{y^2-y-12}{y^2+5y+4}$$

$\frac{(y-1)(y+4)}{(1-y)(y+3)}$

$y \neq -4, -3, -1, 1, \text{or } 4$

3. **Add, state restrictions on the variables.**

$$\frac{4}{x^2-25} + \frac{6}{x^2+6x+5}$$

$\frac{2x(5x-13)}{(x+1)(x-5)(x+5)}$

$x = -1, \pm 5$

4. **Subtract, state restrictions on the variables.**

$$\frac{x-2}{4x+8} - \frac{x+6}{5x+10}$$

$\frac{x-34}{20(x+2)}$



*Jean Adams*

# Concept 3

① $\frac{5x-20}{3x+15} \cdot \frac{7x+35}{5x^2-80}$

$\frac{5(x-4)}{3(x+5)} \cdot \frac{7(x+5)}{5(x^2-16)}$

$\frac{\cancel{5}(x-4)}{3\cancel{(x+5)}} \cdot \frac{7\cancel{(x+5)}}{\cancel{5}(x-4)(x+4)}$

$\frac{7}{3(x+4)}$ $x \neq -5, x \neq \pm 4$

② $\frac{y^2-5y+4}{1-y^2} \div \frac{y^2-y-12}{y^2+5y+4}$

$\frac{(y-1)(y-4)}{(1-y)(1+y)} \div \frac{(y-4)(y+3)}{(y+1)(y+4)}$

$\frac{(y-1)\cancel{(y-4)}}{(1-y)(1+y)} \cdot \frac{(y+1)(y+4)}{\cancel{(y-4)}(y+3)}$

$\frac{(y-1)(y+4)}{(1-y)(y+3)}$ $y \neq 1, -1, -4, 4, \text{or } -3$

# Concept 3

(3) $\frac{4}{x^2-25} + \frac{6}{x^2+6x+5}$

$\frac{(x+1)}{(x+1)}\cdot\frac{4}{(x-5)(x+5)} + \frac{6}{(x+1)(x+5)}\cdot\frac{(x-5)}{(x-5)}$

$\frac{4x+4}{(x+1)(x-5)(x+5)} + \frac{6x-30}{(x+1)(x+5)(x-5)}$

$\frac{10x-26}{(x+1)(x-5)(x+5)}$

$\frac{2(5x-13)}{(x+1)(x-5)(x+5)}$ $x \neq -1, \pm 5$

(4) $\frac{x-2}{4x+8} - \frac{x+6}{5x+10}$

$\frac{5}{5}\cdot\frac{x-2}{4(x+2)} - \frac{x+6}{5(x+2)}\cdot\frac{4}{4}$

$\frac{5x-10}{20(x+2)} - \frac{4x+24}{20(x+2)}$

$\frac{x-34}{20(x+2)}$

# CONCEPT #4:
# Complex Fractions

**Simplify each complex fraction.**

1. $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

$\dfrac{3}{3x+2}$

2. $\dfrac{\frac{5}{x+3}}{2+\frac{1}{x+3}}$

$\dfrac{5}{2x+7}$

3. $\dfrac{\frac{x^2-x-12}{x^2-3x-10}}{\frac{x^2+9x+18}{x^2-5x-14}}$

$\dfrac{5}{2x+7}$ $\dfrac{(x-4)(x-7)}{(x-5)(x+6)}$



*Jean Adams*

# Concept 4

① $\dfrac{\frac{9x}{9x}\cdot\left|-\frac{2}{3x}\cdot\frac{3}{3}\right|}{\frac{9x}{9x}\cdot x-\frac{4}{9x}}$

$\dfrac{\frac{9x}{9x}-\frac{6}{9x}}{\frac{9x^2-4}{9x}}$

$\dfrac{9x-6}{9x}\div\dfrac{9x^2-4}{9x}$

$\dfrac{9x-6}{9x}\cdot\dfrac{9x}{9x^2-4}$

$\dfrac{9x-6}{9x^2-4}\rightarrow\dfrac{3(3x-2)}{(3x-2)(3x+2)}\rightarrow\dfrac{3}{3x+2}$

② $\dfrac{\frac{5}{x+3}}{\frac{x+3}{x+3}\cdot 2+\frac{1}{x+3}}$

$\dfrac{\frac{5}{x+3}}{\frac{2x+6}{x+3}+\frac{1}{x+3}}\rightarrow\dfrac{\frac{5}{x+3}}{\frac{2x+7}{x+3}}\rightarrow\dfrac{5}{2x+7}$

16 Concept 4

(3) $\dfrac{\dfrac{x^2-x-12}{x^2-3x-10}}{\dfrac{x^2+9x+18}{x^2-5x-14}}$

$\dfrac{\dfrac{(x-4)(x+3)}{(x-5)(x+2)}}{\dfrac{(x+6)(x+3)}{(x-7)(x+2)}}$

$\dfrac{(x-4)(x+3)}{(x-5)(x+2)} \cdot \dfrac{(x-7)(x+2)}{(x+6)(x+3)}$

$\dfrac{(x-4)(x-7)}{(x-5)(x+6)}$

# CONCEPT #5:
## Solving Equations

**Solve each equation. If using the Quadratic Formula is necessary, round to two decimal places. Check the solution.**

1. $x - \frac{12}{x} = 1$ $\{-3, 4\}$

2. $\frac{x}{x-2} + \frac{x}{x^2-4} = \frac{x+3}{x+2}$ $\{-3\}$

3. $\frac{2}{x+5} - \frac{1}{x-3} = 3$ $\{-2.63, 4.30\}$



*Jean Adams*

(1) {Concept 5}

$$\frac{x}{x} \cdot x - \frac{12}{x} = 1 \cdot \frac{x}{x}$$

$$\frac{x^2}{x} - \frac{12}{x} = \frac{x}{x}$$

$$x^2 - 12 = x$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \quad x = -3$$

(2) $$\frac{x}{x-2} + \frac{x}{x^2-4} = \frac{x+3}{x+2}$$

$$\frac{(x+2)}{(x+2)}\frac{x}{(x-2)} + \frac{x}{(x+2)(x-2)} = \frac{x+3}{(x+2)} \cdot \frac{(x-2)}{(x-2)}$$

$$\frac{x^2+2x}{(x+2)(x-2)} + \frac{x}{(x+2)(x-2)} = \frac{x^2+x-6}{(x+2)(x-2)}$$

$$x^2 + 3x = x^2 + x - 6$$

$$-x \qquad -x$$

$$2x = -6$$

$$x = -3$$

(3) Concepts

$x^2+2x-15$

$$\frac{(x-3)2}{(x-3)(x+5)} - \frac{1(x+5)}{x-3\cdot(x+5)} = \frac{3\cdot(x-3)(x+5)}{(x-3)(x+5)}$$

$$\frac{2x-6}{(x-3)(x+5)} - \frac{x+5}{(x-3)(x+5)} = \frac{3x^2+6x-45}{(x-3)(x+5)}$$

$$-x \quad -11 = 3x^2+6x-45 \quad -x$$

$$-11 = 3x^2-5x-45$$

$$+11 \qquad +11$$

$$3x^2-5x-34=0$$

$$x = \frac{5 \pm \sqrt{(-5)^2-4(3)(-34)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25+408}}{6}$$

$$x = \frac{5 \pm \sqrt{433}}{6}$$

$$x = \frac{5+\sqrt{433}}{6} \qquad x = \frac{5-\sqrt{433}}{6}$$

$$x = 4.30 \qquad x = -2.63$$