Unit 4 Part 2 Notes

Graphs of Rational Functions - Notes

1. A ______________________ is a function of the form __________, where $p(x)$ and $q(x)$ are polynomial functions in x and $q(x) \neq 0$.

   a. The ______________ of a rational function consists of the values of x for which the denominator $q(x)$ is NOT zero.

   b. Write the domain as "All real numbers except $x = \#$"

2. An ______________________ of a graph is a line to which the graph becomes arbitrarily close as $|x|$ or $|y|$ increases without bound.

   a. In other words, a line at which the graph will approach.

   b. Be sure to write as "__________" for a vertical asymptote and "__________" for a horizontal asymptote.

3. Finding Vertical Asymptotes of Rational Functions

   Let $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ have no common factors.

   a. The graph of f has a vertical asymptote at each ______________ of $q(x)$.

   b. Set the ______________________ equal to zero and solve for x.

4. Note: Vertical asymptotes are also __________________________ and will appear to be a part of the graph, but are NOT.

5. Finding Holes of Rational Functions

   If $p(x)$ and $q(x)$ have a __________________________ c, then

   there is a hole in the graph or a vertical asymptote at ___________.

8.2 – 8.3 Graphs of Rational Functions - Notes

6. Finding Horizontal Asymptotes of Rational Functions

   Let $f(x)=\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ have no common factors.

   The graph of f has, at most, one horizontal asymptote.

   a. If the degree of $p(x)$ is __________ than the degree of $q(x)$,

      then the line ________ is a horizontal asymptote

   b. If the degree of $p(x)$ is __________ to the degree of $q(x)$, then

      the line ________ is a horizontal asymptote, where a is the leading

      coefficient of $p(x)$ and b is the leading coefficient of $q(x)$.

   c. If the degree of $p(x)$ is __________ than the degree of $q(x)$,

      then the graph has _______ horizontal asymptote.

7. Finding Real Zeros/Roots/x-intercepts of Rational Functions
   a. Find real zeros/roots/x-intercepts of rational functions by setting y equal to zero.
   b. Set the ____________________ equal to zero and solve for x.

8. Finding y-intercepts of Rational Functions
   a. Set ____________________ equal to zero; write as $(0,y)$

9. A ____________________ is the graph of a rational function of the

   form $f(x)=\frac{a}{x-h}+k$.

   a. Center: __________
   b. Vertical Asymptote: __________
   c. Horizontal Asymptote: __________

8.2 – 8.3 Graphs of Rational Functions - Notes

Examples: Complete the following table.

Function	*Degree of Numerator*	*Degree of Denominator*	*Horizontal Asymptote*	*y-intercept*	*Vertical Asymptote(s)*	*Root(s)*
1. $y=\frac{4}{x^2+3x-28}$						
2. $y=\frac{-3x}{2x+5}$						
3. $y=\frac{(x+3)(x-1)}{x-4}$						
4. $y=\frac{x-1}{x^2+3}$						
5. $y=\frac{3}{x+5}-2$	X	X				
6. $y=\frac{5}{x}$						

Graphing Rational Expressions

Example 1: Simplify the following. State any restrictions on the variables.

a) $\dfrac{(x+1)(x-5)}{(x-5)(x^2-1)}$

b) $\dfrac{x^2+x-12}{x^2+7x+12}$

Vertical Asymptotes: Where the ______________ of a function equals zero.

Point of Discontinuity: A ____________ in the graph.

Example 2: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x)=\dfrac{x^2-1}{x^2-6x+5}$.

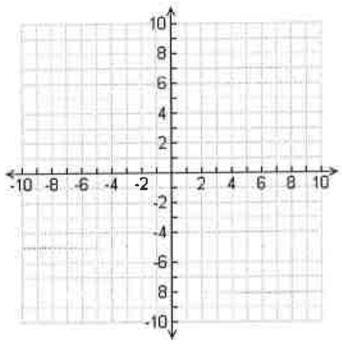
Example 3: Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x)=\dfrac{x^2-4}{x^2+5x+6}$

Horizontal Asymptotes: determined by comparing the degree of the numerator to the degree of the denominator. Let ***m*** = degree of numerator and ***n*** = degree of denominator.

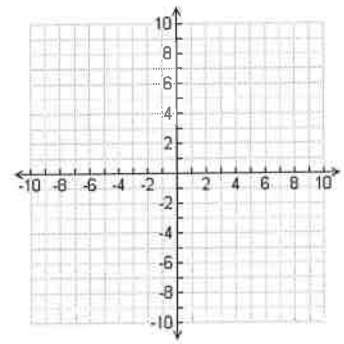
If...	Then the graph has...
$m < n$ $f(x) = \dfrac{x+4}{x^2+5x+4}$	A horizontal asymptote at y = 0 V.A.:____________ Hole(s): ____________ H.A.:____________ Domain:____________
$m = n$ $f(x) = \dfrac{x^2+5x+4}{4x^2-9}$	A horizontal asymptote at the coefficient of m divided by the coefficient of n V.A.:____________ Hole(s): ____________ H.A.:____________ Domain:____________
$m > n$ $f(x) = \dfrac{x^2+5x+4}{x+4}$	No horizontal asymptote V.A.:____________ Hole(s): ____________ H.A.:____________ Domain:____________

Example 4: State the asymptotes and points of discontinuity of each equation, and then graph the function and state the domain.

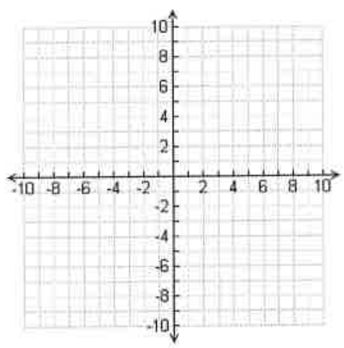
a) $f(x) = \dfrac{x^2 + x - 2}{x - 1}$



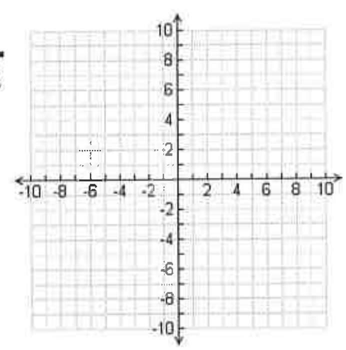
b) $f(x) = \dfrac{2x^2 + 3}{x + 2}$



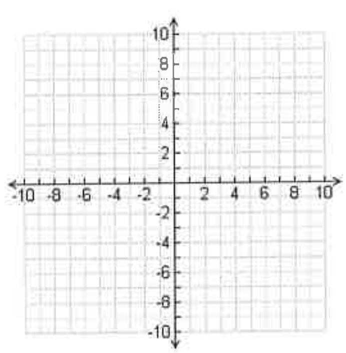
c) $f(x) = \dfrac{x - 1}{x^2 - 1}$



d) $f(x) = \dfrac{x - 3}{x^2 - 7x + 12}$



e) $f(x) = \dfrac{x^2 + 10x + 25}{x^2 + 9x + 20}$



f) $f(x) = \dfrac{x^2 + 12x + 36}{x^2 - 36}$