Multiplying and Dividing Rational Expressions

Example 1: Simplify the following.

a) $\dfrac{(x+1)(x-5)}{(x-5)(x^2-1)}$

b) $\dfrac{x^2+x-12}{x^2+7x+12}$

You Try! Simplify the following.

a) $\dfrac{x^2+6x+9}{x^2-9}$

b) $\dfrac{4x^2+8x}{x^2+6x+8}$

Multiplying Rational Functions

When multiplying rational functions, meaning you are multiplying two factions together, you multiply straight across the top and straight across the bottom, simplifying where you can.

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$

Example 2: Simplify completely.

$$\frac{x^2+2x-8}{x^2+4x+3} \cdot \frac{3x+3}{x-2}$$

Example 3: Simplify Completely.

$$\frac{x^2-9}{x^2+5x+6} \cdot \frac{x+2}{3x-9}$$

You Try! Multiply the following and state the restrictions.

a) $\dfrac{t^2+19t+84}{4t-4} \cdot \dfrac{2t-2}{t^2+9t+14}$

b) $\dfrac{x^2+x-6}{x-5} \bullet \dfrac{x^2-25}{x^2+4x+3}$

Dividing Rational Functions

When dividing rational functions, you multiply the first fraction by the **reciprocal** of the second fraction, simplifying where you can. **SAME-CHANGE-FLIP!**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$$

Example 1: Simplify completely and state the restrictions.

$$\frac{a+2}{a+3} \div \frac{a^2+a-12}{a^2-9}$$

Example 2: Simplify Completely. State all restrictions. $\dfrac{\dfrac{b^2}{25a^2-b^2}}{\dfrac{b}{5r-b}}$

You Try! Divide the following. Be sure to state all restrictions.

a) $\dfrac{\dfrac{-12b+18}{b^2-25}}{\dfrac{4b-6}{b^2-3b-10}}$

b) $\dfrac{3x+12}{2x+4} \div \dfrac{x^2-16}{x+2}$

Adding and Subtracting Rational Expressions

In order to add or subtract fractions, we must first find the _____ _____ _____, or LCD.

a) $\frac{1}{3}+\frac{3}{4}$

b) $\frac{5}{2}-\frac{3}{4}$

Monomial Denominators-FIND A COMMON DENOMINATOR!

- determine what each denominator has that the other denominator is missing
- multiply top and bottom by whatever is missing-to give you the common denominator

Example 1: $\frac{1}{2x}+\frac{1}{2x}$

Example 2: $\frac{-2}{x}-\frac{1}{x}$

Example 3: $\frac{1}{6x}+\frac{2}{3x}-\frac{3}{4x}$

Example 4: $\frac{5y+2}{xy^2}+\frac{2x-4}{4xy}$

Example 5: $\frac{3}{7x^2y}+\frac{4}{21xy^2}$

Example 6: $\frac{3}{8x^3y^3}-\frac{1}{4xy}$

Monomial Denominators – FACTOR & FIND A COMMON DENOMINATOR!

- Always start by factoring polynomial denominators
- Multiply top and bottom by whatever is missing and then combine the numerators

Example 7: $\frac{w+12}{4w-16} - \frac{w+4}{2w-8}$

Example 8: $\frac{y}{2y+4} - \frac{3}{y+2}$

Example 9: $\frac{-3x}{x^2-9} + \frac{4}{2x-6}$

Example 10: $\frac{2x}{x^2-x-2} - \frac{4x}{x^2-3x+2}$

Example 11: $\frac{5x}{x^2-x-6} - \frac{4}{x^2+4x+4}$

Example 12: $\frac{x}{x-1} + \frac{2x-1}{x^2-3x+2}$

Solving Rational Equations

How to Solve Rational Equations

1. Find the least common denominator
2. Write each part of the equation as an equivalent fraction with the least common denominator
3. Multiply all parts of the equation by the LCD to remove fractions
4. Solve.
5. Check your answers

How to Solve Rational Equations that are Proportions

1. Cross Multiply
2. Solve
3. Check your answers

Example 1: Solve $\frac{y}{5}+\frac{y}{2}=7$

Example 2: Solve $\frac{3x-2}{12}-\frac{1}{6}=\frac{1}{6}$

Example 3: Solve $\frac{11}{3x}-\frac{1}{3}=\frac{-4}{x^2}$

Example 4: Solve $\frac{5}{2x}-\frac{2}{3}=\frac{1}{x}+\frac{5}{6}$

Example 5: Solve $x+\frac{6}{x}=-5$

Example 6: Solve $\frac{5x-2}{x-4}=-3$

Example 7: Tim and Ashley mow lawns together. Tim working alone could complete the job in 4.5 hours, and Ashley could complete it alone in 3.7 hours. How long does it take to complete the job when they work together?

$$\frac{timetogether}{timeofperson\#1}+\frac{timetogether}{timeofperson\#2}=1$$

Directions: Simplify each rational expression. State all restrictions.

1. $\dfrac{8x^2-10x+3}{6x^2+3x-3}$

2. $\dfrac{5x}{2y+4}-\dfrac{6}{y^2+2y}$

3. $\dfrac{x^2-16}{2x+8}\div\dfrac{(x-4)^2}{8x-32}$

4. $3x-\dfrac{x^2-5x}{x^2-2}$

5. $\dfrac{3x-6}{5x-20}\cdot\dfrac{x-8}{5x-10}$

6. $\dfrac{y^2-25}{y^2-16}\div\dfrac{2y+10}{y^2-4y}$

7. $\dfrac{8}{3x^3y}+\dfrac{4}{9xy^3}$

8. $\dfrac{x^2}{x^2+2x+1}\div\dfrac{3x}{x^2-1}$

9. $\dfrac{7}{5y+25}-\dfrac{4}{3y+15}$

Directions: Solve each equation. Check your solutions.

10. $\frac{x}{3}+\frac{x}{2}=10$

11. $\frac{-2}{x^2-2}=\frac{2}{x-4}$

12. $\frac{1}{2x}-\frac{2}{5x}=\frac{1}{2}$

13. $\frac{3}{1-x}=\frac{2}{1+x}$

14. $\frac{1}{x}-\frac{1}{6}=\frac{4}{3x^2}$

15. $\frac{x+3}{x^2+3x-4}=\frac{x+2}{x^2-16}$

16. Becky and Kendra start a business painting fences. They can paint 200 ft^2 of fence in 40 min if they work together. If Kendra paints four times faster than Becky, how long would it take each of them to paint a 500-ft^2 fence working alone?