3rd

AFM review Sequences and Series

1. Write the 1st 4 terms of the sequence:

a) $a_n = 4^{(n-1)}$ 1, 4, 16, 64

b) $a_n = 8 + 6(n-1)$ 8, 14, 20, 26

c) $a_n = (-1)^n (n+3)$ $-4, 5, -6, 7$

2. Find the 20th term of the arithmetic sequence given:

a) $a_1 = 3$, $d = 7$ $T_N = 3+(n-1)\cdot 7$ $T_{20} = 3+(20-1)\cdot 7$ $T_{20} = 136$

b) $a_1 = -1$, $d = 6$ $T_N = -1+(n-1)\cdot 6$ $T_{20} = -1+(20-1)\cdot 6$ $T_{20} = 113$

3. Write the 1st 4 terms of the arithmetic sequence:

a) $a_n = a_{n-1} + 6$, $a_1 = -4$ $-4, 2, 8, 14$

b) $a_n = a_{n-1} - 1.2$, $a_1 = 6$ $6, 4.8, 3.6, 2.4$

4. Write the explicit formula and a_{20} for these sequences

a) 5, 12, 19, 26, ... $T_N = 5+(n-1)\cdot 7$ $T_{20} = 5+(20-1)\cdot 7$ $T_{20} = 138$

b) $a_n = a_{n-1} + 3$, $a_1 = 4$ $T_N = 4+(n-1)\cdot 3$ $T_{20} = 4+(20-1)\cdot 3$ $T_{20} = 61$

5. Write the first five terms of the geometric sequence whose first term is 2, and whose common ratio is 2.

2, 4, 8, 16, 32

6. Write the first 5 terms of the geometric sequence.

a) $a_n = 6(a_{n-1})$, $a_1 = -4$ $-4, -24, -144, -864, -5,184$

b) $a_n = 3(a_{n-1})$, $a_1 = 7$ $7(3)^{n-1}$ $7, 21, 63, 189, 567$

7. Given an Arithmetic Sequence with $a_7 = 46$ and $a_{11} = 74$ find:

$a_1 =$ 4 $d =$ 7 $\frac{46-74}{7-11} = \frac{-28}{-4} = 7$

Explicit Definition $T_N = 4+(n-1)\cdot 7$ Recursive Definition $a_1 = 4$ $T_N = a_{N-1} + 7$

8. Given a Geometric Sequence with $a_1 = 2$ and $a_5 = 162$ find

$a_1 =$ 2 $r =$ 3 $A_N = a_1 \cdot r^{n-1}$ $162 = 2 \cdot r^{5-1}$ $\frac{162}{2} = \frac{2 \cdot r^4}{2}$ $\sqrt[4]{81} = \sqrt[4]{r^4}$ $r = 3$

Explicit Definition $A_N = 2(3)^{n-1}$ Recursive Definition $a_1 = 2$ $A_N = a_{N-1} \cdot 3$

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c) $a_n = (-1)^n (n + 3)$ $-4, 5, -6, 7$

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$T_N = 3+(n-1)\cdot 7$

$T_{20} = 3+(20-1)\cdot 7$

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8. Given a Geometric Sequence with $a_1 = 2$ and $a_5 = 162$ find

$a_1 = 2$ $r = 3$

$A_N = a_1 \cdot r^{n-1}$

$162 = 2 \cdot r^{5-1}$

$\frac{162}{2} = \frac{2 \cdot r^4}{2}$

$\sqrt[4]{81} = \sqrt[4]{r^4}$

$r = 3$

Explicit Definition $A_N = 2(3)^{n-1}$ Recursive Definition $a_1 = 2$ $A_N = a_{N-1} \cdot 3$

9. Given the sequence 5, 9, 13, 17… Find the 20th term.

81

10. Given the arithmetic sequence with $a_{15} = 38$ and $d = -3$, find a_1.

$T_N = a_1 + (N-1) \cdot -3$

$38 = a_1 + (15-1) \cdot -3$

$38 = a_1 + (14) \cdot -3$

$38 = a_1 - 42$

$80 = a_1$

11. If a sequence is arithmetic Find n if $a_n = 633$, $a_1 = 9$, and $d = 24$

$633 = 9 + (N-1) \cdot 24$

$624 = (N-1) \cdot 24$

$N - 1 = 26$

$N = 27$

12. Find the sum of the first 30 terms of the sequence 1, 9, 17, 25…

+8 +8 +8

Arithmetic $S_n = \frac{n}{2}(a_1 + a_n)$ $a_{30} = 1 + (30-1) \cdot 8$

$S_{30} = \frac{30}{2}(1 + 233)$

$S_{30} = 3510$

13 Write out the first three terms and the last term of the sequence. Then use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum.

$S_N = \frac{n}{2}(a_1 + a_n) \sum_{i=1}^{20} 5i$

1st 2nd 3rd

5, 10, 15 …… 20

100

$S_{20} = \frac{20}{2}(5 + 100)$

$S_{20} = 1050$

14. . Find the sum of the first 11 terms of the geometric sequence shown below.

4, 12, 36, 108, . . .

$S_N = \frac{a_1(1 - r^N)}{1 - r}$

$a_1 = 4$ $r = 3$ $N = 11$

$S_{11} = \frac{4(1 - 3^{11})}{(1 - 3)}$

$S_{11} = 354{,}292$

15. Evaluate.

G $\sum_{i=2}^{5} 3^i$

$S_n = \frac{a_1(1 - r^N)}{1 - r}$

$S_5 = \frac{9(1 - 3^4)}{(1 - 3)}$

$S_5 = 360$

16. . Find the indicated sum. Use the formula for the sum of the first n terms of a geometric sequence.

G $\sum_{i=1}^{10} 4 \cdot 2^i$

$S_N = \frac{a_1(1 - r^N)}{(1 - r)}$

17. $a_1 = 8$ $r = 2$ $N = 10$ $S_{10} = \frac{8(1 - 2^{10})}{(1 - 2)}$

$S_{10} = 8184$

Find the sum of the infinite geometric series.

smaller

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots$ $S = \frac{a_1}{1 - r} = \frac{1}{1 - \frac{1}{2}} = 2$

18 Find the sum of the infinite geometric series.

smaller

$\frac{3}{7^0} + \frac{3}{7} + \frac{3}{7^2} + \frac{3}{7^3} + \cdots$ $= S = \frac{3}{\left(1 - \frac{1}{7}\right)} = 3.5$

. In an Arithmetic Series find the sum of the 1st 20 terms if the 1st term is -4 and the common difference is 6.

$n = 20$

$a_1 = -4$

$a_{20} = -4 + (20-1) \cdot 6$

$a_{20} = 110$

$\frac{20}{2}(-4 + 110) = 1060$

In an infinite geometric series $a_1 = 1$ and $S = 2.5$. Find the common ratio.

$2.5 = \frac{1}{1 - r}$

$2.5 - 2.5r = 1$

$-2.5r = -1.5$

$r = 3/5$

Name_____ Date_____ Period_____

SEQUENCES & SERIES WORKSHEET

Arithmetic: $a_n = a_1 + (n-1)d$ $\quad S_n = \frac{n(a_1 + a_n)}{2}$

Geometric: $a_n = a_1 r^{n-1}$ $\quad S_n = \frac{a_1(1-r^n)}{1-r}$

Use the formulas provided to you to complete the following. Determine what type of sequence the following are and then complete the problem.

1. a=-5, d=4, n=9; find the nth term

A $a_N = -5 + (n-1) \cdot 4$

$a_9 = -5 + (9-1) \cdot 4$

$a_9 = 27$

2. a=5, n=4, r=3; find the nth term

G $a_N = 5(3)^{n-1}$

$a_4 = 5(3)^{4-1}$

$a_4 = 135$

3. a=3, d=-4, n=6; find the nth term

A $a_N = 3 + (n-1) \cdot 4$

$a_6 = 3 + (6-1) \cdot -4$

$a_6 = -17$

4. a=-4, n=6, r=-2; find the nth term

G $a_N = -4(-2)^{n-1}$

$a_6 = -4(-2)^{6-1}$

$a_6 = 128$

Find the missing terms in each sequence. You are given what type of sequence represents each one.

5. $\frac{2}{9}$ (1st), $\frac{2}{3}$ (2nd), 2 (3rd), $\frac{6}{4}$, $\frac{18}{5}$, 54 (geometric)

(5) $a_N = a_1 \cdot r^{N-1}$

$54 = 2r^{(6-3)}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$r = 3$

6. 3 (1st), $\frac{26}{3}$, $\frac{43}{3}$, 20 (arithmetic)

(6) $a_N = a_1 + (n-1)d$

$20 = 3 + (4-1)d$

$17 = 3d$

$\frac{17}{3} = d$

$\frac{20-3}{4-1} = \frac{17}{3}$

7. 5, $\frac{37}{3}$, $\frac{59}{3}$, 27 (arithmetic) → $\frac{27-5}{4-1} = \frac{22}{3}$

8. 32 (1st), 48, 72, 108 (5th), 162 (geometric) → $162 = 32 \cdot r^{5-1}$

$\sqrt[4]{\frac{81}{16}} = \sqrt[4]{r^4}$ $\quad r = \frac{3}{2}$

9. -16 (1st), -10, -4 (2nd), 2 (3rd), 8 (6th), 14 (arithmetic) → $14 = -10 + (6-2) \cdot d$

$24 = 4d$

$6 = d$

10. Find the 15th term for the arithmetic sequence –3, 3, 9, …

$a_N = -3 + (n-1) \cdot 6$

$a_{15} = -3 + (15-1) \cdot 6$

$a_{15} = 81$

11. Find the first 4 terms of the geometric sequence with $a = -6$ and $r = -\frac{2}{3}$

$a_N = -6(-\frac{2}{3})^{n-1}$

$a_4 = -6(-\frac{2}{3})^{4-1}$

$a_4 = \frac{16}{9}$

Find S_n for each series described. You will need to determine if the series is arithmetic or geometric.

12. $160 + 80 + 40 + \ldots$, $n = 6$

$\div 1/2$

$S_n = \frac{a_1(1-r^N)}{(1-r)}$

$S_6 = \frac{160\left(1-\frac{1}{2}^6\right)}{(1-1/2)}$

$S_6 = 315$

13. $a = 5$, $r = -\frac{1}{2}$, $n = 7$

$S_n = \frac{a_1(1-r^N)}{1-r}$

$S_7 = \frac{5\left(1-\left(-\frac{1}{2}\right)^7\right)}{(1-(-1/2))}$

$S_7 = \frac{85}{64}$

14. $a = 13$, $d = -6$, $n = 21$

$a_{21} = 13 + (21-1)\cdot -6$

$a_{21} = -107$

$S_N = \frac{n}{2}(a_1 + a_N)$

$S_{21} = \frac{21}{2}(13 + -107)$

$S_{21} = -987$

15. $d = -\frac{2}{3}$, $n = 16$, $a_n = 44$

$S_N = \frac{n}{2}(a_1 + a_2)$

$S_{16} = \frac{16}{2}(54 + 44)$

$S_{16} = 784$

$44 = a_1 + (16-1)\left(\frac{2}{3}\right)$

$44 = a_1 + 15\left(\frac{2}{3}\right)$

$44 = a_1 - 10$

$54 = a_1$

Find "a" for each geometric series.

16. $S_n = -55$, $r = -\frac{2}{3}$, $n = 5$

$S_n = \frac{a_1(1-r^N)}{1-r}$

$-55 = \frac{a_1\left(1-\left(-\frac{2}{3}\right)^5\right)}{(1-(-2/3))}$

$-55 = \frac{275}{243}a_1$

$5/3$

$-275 = 275a_1$

$-1 = a_1$

17. $S_n = 2457$, $a = 3072$, $r = -4$

$S_n = \frac{a_1(1-r^N)}{1-r}$

$2457 = \frac{3072(1-(-4)^N)}{(1-(-4))}$

$2457 = \frac{3072(1-(-4)^N)}{5}$

$12285 = 3072(1-(-4)^N)$

Find the first 3 terms of each arithmetic series.

18. $a = 14$, $a_n = -85$, $S_n = -1207$

$S_n = \frac{n}{2}(a_1 + a_n)$

$-1207 = \frac{n}{2}(14 + -85)$

$-1207 = \frac{n}{2}(-71)$

$-2414 = -71n$

$34 = n$

19. $n = 16$, $a_n = 15$, $S_n = -120$

$S_N = \frac{n}{2}(a_1 + a_2)$

$-120 = \frac{16}{2}(a_1 + 15)$

$-120 = 8(a_1 + 15)$

$-15 = a_1 + 15$

$-30 = a_1$